CS 4510

Turing Machine Lower Bounds

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1. We will walk together through the proof that it takes $\Theta(n^2)$ time on a one tape DTM to decide $PAL = \{x \in \Sigma^* \mid x = x^R\}$. This is a beautiful theorem. The result basically says you cannot do better than the obvious way up to constant factors.

   (a) (5 pts) Prove you can decide PAL in $O(n^2)$ on a one tape DTM

   Your answer here

   (b) (10 pts) Suppose some M decides PAL, and consider its execution on input $w0^nw^R$ such that $|w| = n$. The middle third of this string is all zeroes. M says $w0^nw^R \in PAL$ and terminates after $T(x)$ steps. Let the tape cells be numbered 1,2,3,... and notice that the cells containing the middle third zeroes are numbered from $(n+1), ..., 2n$.

   Prove that there exists some i with $n+1 \leq i \leq 2n$ such that the i'th cell is crossed over by the tape head $\leq T(x)/n$ times.

   Your answer here

   (c) (5 pts) Let m be the number of times this i'th cell is crossed over, so $m \leq T(x)/n$. Consider the sequence of pairs, of states and symbols $(q_1, a_1), (q_2, a_2), ..., (q_m, a_m)$ such that the first time the head crossed our cell, it was in state q_1 and read a_1, second time it was in state q_2 and read a_2, and so on. This is for any $q_j \in Q$ and $a_j \in \Gamma$.

   Prove that if j is odd, then (q_j, a_j) was a right move, and if j was even, then (q_j, a_j) was a left move.

   Your answer here

   (d) (50 pts) We can describe the string w of $w0^nw^R$ uniquely. Consider the TM M'

   M': Your answer here

   Prove that M' and the sequence $(q_1, a_1), ..., (q_2, a_2)$ uniquely describe w. (Hint: Suppose M' could print some $y \neq w$. What does this tell you about the behavior of M on $y0^nw^R$?)

   Your answer here

   (e) (10 pts) Prove that $K(w) \leq |\langle M' \rangle| + |\langle (q_1, a_1), ..., (q_m, a_m) \rangle| + 2\log n$

   Your answer here

   (f) (5 pts) Prove that $K(w) \leq |\langle M' \rangle| + mc + 2\log n$ for some constant c

   Your answer here

   (g) (5 pts) Prove that $K(w) \leq |\langle M' \rangle| + cT(n)/n + 2\log n$

   Your answer here

   (h) (10 pts) Conclude $T(x) = \Omega(n^2)$

   Your answer here