

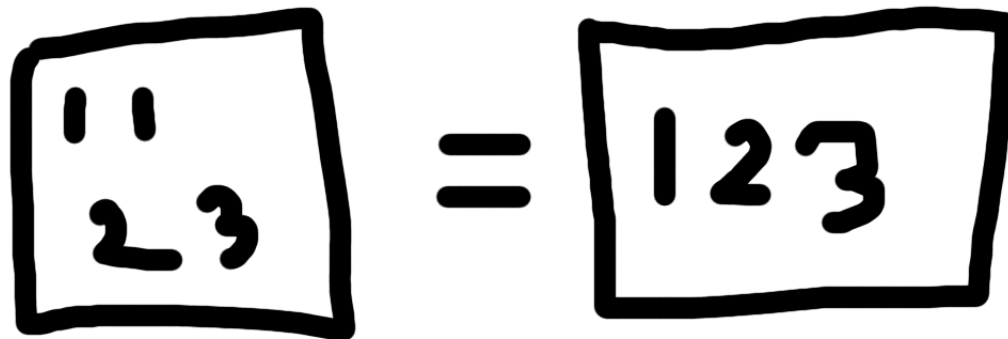
## MATHEMATICAL PROOFS OUTLINE OF BOOK

1. Set Theory
  - (a) Sets
  - (b) Subsets
  - (c) Set Operations
  - (d) Cartesian Product
  - (e) Partition of Sets
  - (f) a note on infinite sets
2. Formal Logic
  - (a) Logic
  - (b) Ands and Ors
  - (c) Truth Tables
  - (d) implication
  - (e) biconditionals
  - (f) existential quantifiers
  - (g) tautologies and contradictions
  - (h) a note on empiricism
3. Direct and Contrapositive Proofs
  - (a) direct proof
  - (b) contrapositive proof
  - (c) proof by cases
  - (d) integers
  - (e) real numbers
  - (f) sets
  - (g) one more proof section
4. Existence and Contradiction
  - (a) counterexample
  - (b) contradiction
  - (c) non constructive proofs

5. Induction
  - (a) Principle of Mathematical Induction
  - (b) Generalized version
  - (c) Strong version
6. The Pigeonhole Principle
  - (a) Pigeons!
  - (b) Stronger form
  - (c) Generalized form
  - (d) a note on infinite sets (again)
7. Probabilistic Proof
  - (a) Certainty and examples.
8. Extra fun proofs
  - (a) proofs in geometry
  - (b) proofs in graph theory
  - (c) proofs in computer science
  - (d) proofs in game theory

### **Sets**

Man what are like, sets anyway? well its like a bunch of stuff together, thats also a thing. You know? so like you have a bunch of things in a box. and this box is also a thing that could be a stuff. The things in a set are called elements. You can only have one kind of element in a set, per set. For example, lets take some household items as shown here:



So these boxes are our sets, which contain a bar of soap, a towel, and the cursed suit of armor that if worn will release the volcano god arkantaks-dasdlk. Notice one of the boxes has two bars of soap, but it doesn't really matter, and is the same as if it had one bar of soap.

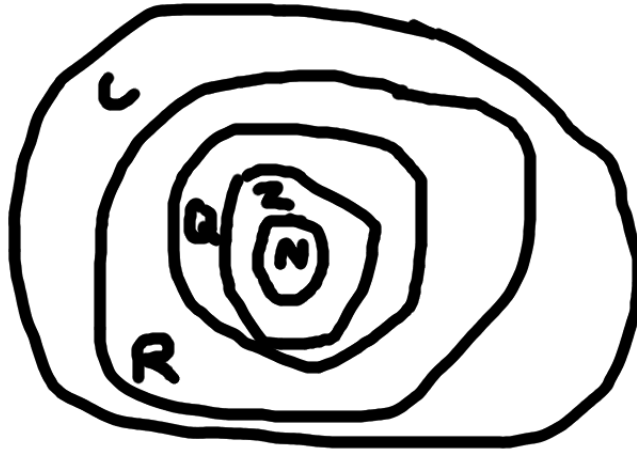
As another example, how about the set of counting numbers? You know that the number one is a number, since you are holding one copy of this book (probably), and the number two is a number because just trust me okay? This is a set known as the *Natural Numbers*, denoted with a  $\mathbb{N}$ . Where  $\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$ . (Mathematicians like to use big letters since they have big egos.) The number 1 is in the set, so we can write this as  $1 \in \mathbb{N}$ . Notice zero, or any negative number is not an element of this set. We can write this as  $0 \notin \mathbb{N}$

Now what about the set with zero and the negative numbers? This set is the the set of Integers. Its denoted with a  $\mathbb{Z}$ . It comes from Zahl, the german word for integer. This time,  $\mathbb{Z} = \{\dots - 3, -2, -1, 0, 1, 2, 3, \dots\}$

What about numbers like  $\frac{3}{4}$ ? or  $\frac{17}{38}$ ? Those exist inside a set called the rational numbers, denoted with a  $\mathbb{Q}$ . Where,  $\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z} \text{ and } b \neq 0\}$  This is another way to write a set, specify all elements of a set look a certain way.

Irrational numbers, transcendental numbers, and others, these all live inside the set of all real numbers, denoted with a  $\mathbb{R}$ .

You would think this would be all the numbers, but no! Notice the number  $\sqrt{-1}$  is not a real number (where would it be on the real number line?) so now we have an even bigger set called the *Complex Numbers* denoted with a  $\mathbb{C}$  such that  $\mathbb{C} = \{a + bi \mid i = \sqrt{-1} \text{ and } a, b \in \mathbb{R}\}$



### Subsets

A set, lets call it  $A$ , is a subset of another set, denoted  $B$ , if every element of  $A$  is also an element of  $B$ . For example, if  $A = \{1, 2, 3\}$  and  $B = \{2, 3\}$ , then  $B \subseteq A$ . If every element of  $B$  is also an element of  $A$ , and there is atleast one element of  $A$  not in  $B$ , then we can say  $B$  is a *proper subset* of  $A$ ;  $B \subset A$ .

something something transitivity of subsetness

We can use set containment to show that two sets are equal. If every element in a set  $A$  is also an element in a set  $B$ , and every element in a set  $B$  is also an element in a set  $A$ , then these set are equal. If  $A \subseteq B$  and  $A \supseteq B$ , then  $A = B$

Something something null set

Something something powerset

### Set Operations

thing about union

thing about venn diagrams

thing about disjoint

thing about subtraction

thingabout complement  
thingabout mega union  
thingabout mega disjoint

### **Partitions of Sets**

definition  
diagram  
two examples

### **Note on infinite sets**

cantors array counterexample  
countably infinite  
uncountably infinite